

Supergravity Duals for N=2 Gauge Theories

Arvind Rajaraman *

Department of Physics and Astronomy, Rutgers University, Piscataway, NJ 08855.

Abstract

We construct supergravity solutions for Dp-branes at orbifold points. The solutions are written in terms of a single function, which is the solution to a nonlinear differential equation. The near horizon limits of these solutions are dual, in the AdS/CFT sense, to super-Yang-Mills theories with 8 supercharges in various dimensions. In particular, we present a dual to $\mathcal{N} = 2$ $SU(N)$ SYM theory in 3+1 dimensions, and analyse some aspects of the duality.

*e-mail address: arvindra@muon.rutgers.edu

I. INTRODUCTION

The AdS/CFT correspondence [1], which provides a supergravity dual for $\mathcal{N} = 4$ super-Yang-Mills theories, can be generalized to generate supergravity duals for theories with less supersymmetry. In particular, there has been much interesting recent work on duals to $\mathcal{N} = 1$ gauge theories (including $\mathcal{N} = 1$ $SU(N)$ super-Yang-Mills theory) [2–7]. However, $\mathcal{N} = 2$ theories have received less attention.

In this paper, we will construct supergravity duals to $SU(N)$ super Yang-Mills theories with 8 supercharges in various dimensions. This is closely related to the work of [8], where supergravity solutions corresponding to D4-branes ending on NS5-branes were found. (This would be a dual to MQCD, which is in the same universality class as super-Yang-Mills theory [9]).

We shall here construct solutions corresponding to branes on orbifolds. These solutions are complicated; nevertheless, they can be found by the methods discussed in [10]. These solutions are determined by one function, which satisfies a nonlinear differential equation. We have not found an explicit solution to this differential equation, nevertheless, the solution is determined in principle.

Branes on orbifolds were studied by Douglas and Moore [11] (see also [12]) who found a general prescription for the worldvolume theory of these branes. In particular, N D5-branes wrapped on a 2-cycle of a T^4/Z_N orbifold have a worldvolume theory which is precisely 4-dimensional $SU(N)$ super Yang-Mills theory with 8 supercharges.

We can now follow the general reasoning of Maldacena. We can find the supergravity solution produced by these D5-branes. The near horizon limit of this geometry is then dual to the worldvolume theory, i.e. it is dual to 4-dimensional $\mathcal{N} = 2$ super Yang-Mills theory.

We shall start with a D2-brane on a T^4/Z_N orbifold. It will prove extremely helpful to consider not the singular limit, but a (partially) resolved orbifold where one two-cycle has been resolved. We shall look at a D2-brane wrapped on this resolved 2-cycle.

The metric for the resolved orbifold is known exactly to be

$$ds^2 = H^{-1}(dx_9 + A_7 dx^7 + A_8 dx^8)^2 + H(dx_7^2 + dx_8^2 + dx_6^2) \quad (1)$$

with

$$\begin{aligned} \partial_6 A_8 &= \partial_7 H & \partial_6 A_7 &= -\partial_8 H \\ \partial_7 A_8 - \partial_8 A_7 &= -\partial_6 H \end{aligned}$$

(We have chosen an unconventional gauge choice where $A_6 = 0$. Retaining A_6 leads to a more symmetric set of equations: see, for example [12].)

Here H is a harmonic function satisfying $(\partial_6^2 + \partial_7^2 + \partial_8^2)H = 0$. If we want to resolve one 2-cycle in a Z_N orbifold, we take

$$H = \frac{N_1}{x_6^2 + x_7^2 + x_8^2} + \frac{N_2}{(x_6 - a)^2 + x_7^2 + x_8^2} \quad (2)$$

A 2-brane can then extend between the centres at $x_6 = 0$ and $x_6 = a$, and the second worldvolume direction of the 2-brane wraps the x_9 direction. (There is a periodic identification of x_9 with period 4π .)

The metric for the resolved orbifold is clearly very similar to the metric of parallel 5-branes. Similarly the 2-brane wrapping the two cycle is very similar to a brane stretching between these parallel 5-branes. The supergravity solution can thus be found using the methods described in [10] for the construction of intersecting brane solutions. We shall describe this construction in great detail in the next section.

Hoever, the orbifold point also has a nonzero B-field turned on [13]. This B-field induces a D0-brane charge on the D2-brane. We should therefore look for a solution with both D2-brane and D0-brane charge. This can be done by lifting the D2-brane solution to 11 dimensions, and boosting it, thereby adding D0-brane charge. By an appropriate choice of the boost parameter, we can tune the B-field on the 2-cycle to any value desired.

The solutions for the other branes on the orbifold can be found by T-duality. In particular, T-dualizing thrice, we get a D5-brane wrapped on the orbifold 2-cycle, which as we have seen, is dual to 3+1 dimensional $\mathcal{N} = 2$ SYM.

The solution we find is different from other attempts to realize $\mathcal{N} = 2$ theories [14]. The difference is that in other cases, the full theory is not exactly $SU(N)$ $\mathcal{N} = 2$ super-Yang-Mills theory in 3+1 dimensions. In some cases, the theory is regulated in the UV by a 6-dimensional CFT, and in other cases, it is regulated by a theory with a larger gauge group in 4-dimensions. Furthermore, some of these theories live on $S^3 \times R$ rather than $R^{3,1}$. The construction we present is dual to the exact $\mathcal{N} = 2$ super-Yang-Mills theory on $R^{3,1}$ with gauge group $SU(N)$. This is why it differs from the other constructions.

II. 2-BRANE ON Z_N ORBIFOLD

A. Notation

We will start by constructing the supergravity solution for a D2-brane wrapped on a 2-cycle of a Z_N orbifold. We can approach the SUGRA solution of this system in the same way as [10]. First we establish some notation.

We will denote the directions x_1, x_2, x_3, x_4, x_5 collectively by x_a . The directions x_7, x_8 will collectively be called x_α .

The approach in [10] used an analysis of the condition for conserved supercharges

$$\partial_\mu \epsilon + \frac{1}{2} \omega_\mu^{ab} \gamma_{ab} \epsilon + (c_1 e^\Phi F_\mu^{abc} \gamma_{abc} + c_2 e^\Phi \Phi_{abcd} g_\mu^{abcd}) \epsilon = 0 \quad (3)$$

where c_1, c_2 are constants with $c_1 = -c_2$.

We will make the following ansatz (see [10] for a more detailed discussion)

- $\epsilon = (g_{00})^{\frac{1}{4}} \epsilon_0$ where ϵ_0 is a constant spinor.
- The constant spinor satisfies

$$(1 - \gamma^{069}) \epsilon_0 = (1 - \gamma^{6789}) \epsilon_0 = 0 \quad (4)$$

- The metric components will be taken to be diagonal with the addition of $e_{9\tilde{7}}, e_{9\tilde{8}}$ (present already in (1)) and $e_{6\tilde{a}}$ (induced by the 2brane).

- The nonzero gauge field strengths are

$$G_{069a} \quad G_{069\alpha} \quad G_{09a\alpha} \quad H_{078a} \quad H_{0678} \quad H_{06a\alpha}$$

As in [10], we have chosen to denote the same field strength by different letters depending on the indices. Both G and H are the field strength coupling to $D2$ branes.

B. Supersymmetry equations

The SUSY equations (3) now reduce to a set of algebraic equations which determine the field strengths in terms of the spin connections and also impose some constraints on the spin connections.

To figure out these algebraic equations, note that the SUSY constraints (4) are singlets under rotations in x_a, x_α . Hence we can decompose the equations in representations of the rotations in x_a, x_α .

For instance, the field strengths and spin connections which transform under rotations in x_1 , but are singlets under other rotations, are ω_A^{A1}, G_{0691} , and H_{0781} . Hence these terms must cancel against each other in each SUSY equation (3) with any μ . For $\mu = 0, 4, 6, 7, 9$, we find respectively

$$\begin{aligned} \omega_0^{01}\gamma_1 + e^\Phi G_{0691}\gamma^{0691} + e^\Phi H_{0781}\gamma^{0781} &= 0 \\ \omega_4^{41}\gamma_1 - e^\Phi G_{0691}\gamma^{0691} - e^\Phi H_{0781}\gamma^{0781} &= 0 \\ \omega_6^{61}\gamma_1 + e^\Phi G_{0691}\gamma^{0691} - e^\Phi H_{0781}\gamma^{0781} &= 0 \\ \omega_7^{71}\gamma_1 - e^\Phi G_{0691}\gamma^{0691} + e^\Phi H_{0781}\gamma^{0781} &= 0 \\ \omega_9^{91}\gamma_1 + e^\Phi G_{0691}\gamma^{0691} - e^\Phi H_{0781}\gamma^{0781} &= 0 \end{aligned} \tag{5}$$

We have chosen a convenient normalization where c_1 is absorbed into the field strengths.

From these equations we find

$$e^\Phi G_{0691}\gamma^{0691} = -\frac{1}{2}(\omega_0^{01} - \omega_7^{71})\gamma_1 \tag{6}$$

$$e^\Phi H_{0781}\gamma^{0781} = -\frac{1}{2}(\omega_0^{01} + \omega_7^{71})\gamma_1 \tag{7}$$

and the constraints

$$\omega_4^{41} + \omega_0^{01} = 0 \quad (8)$$

$$\omega_6^{61} + \omega_7^{71} = 0 \quad (9)$$

$$\omega_9^{91} + \omega_7^{71} = 0 \quad (10)$$

We can similarly write down the remaining algebraic equations.

$$\begin{aligned} \gamma^{0971} G_{0971} + \gamma^{0681} H_{0681} &= 0 \\ \omega_1^{67} \gamma_{67}^1 + \omega_1^{89} \gamma_{89}^1 + e^\Phi G_{0971} \gamma^{0971} + e^\Phi H_{0681} \gamma^{0681} &= 0 \\ \omega_6^{17} \gamma_{17}^6 - e^\Phi G_{0971} \gamma^{0971} + e^\Phi H_{0681} \gamma^{0681} &= 0 \\ \omega_7^{16} \gamma_{16}^7 + e^\Phi G_{0971} \gamma^{0971} - e^\Phi H_{0681} \gamma^{0681} &= 0 \\ \omega_8^{19} \gamma_{19}^8 - e^\Phi G_{0971} \gamma^{0971} + e^\Phi H_{0681} \gamma^{0681} &= 0 \\ \omega_9^{18} \gamma_{18}^9 + e^\Phi G_{0971} \gamma^{0971} - e^\Phi H_{0681} \gamma^{0681} &= 0 \\ \omega_0^{07} \gamma_7 + e^\Phi G_{0697} \gamma^{0697} &= 0 \\ \omega_a^{a7} \gamma_7 - e^\Phi G_{0697} \gamma^{0697} &= 0 \\ \omega_6^{67} \gamma_7 + \omega_6^{98} \gamma_{98}^6 + e^\Phi G_{0697} \gamma^{0697} &= 0 \\ \omega_8^{87} \gamma_7 + \omega_8^{96} \gamma_{96}^8 - e^\Phi G_{0697} \gamma^{0697} &= 0 \\ \omega_9^{97} \gamma_7 + \omega_9^{68} \gamma_{68}^9 + e^\Phi G_{0697} \gamma^{0697} &= 0 \\ \omega_0^{06} \gamma_6 + e^\Phi H_{0678} \gamma^{0678} &= 0 \\ \omega_a^{a6} \gamma_6 - e^\Phi H_{0678} \gamma^{0678} &= 0 \\ \omega_7^{76} \gamma_6 + \omega_7^{89} \gamma_{89}^7 + e^\Phi H_{0678} \gamma^{0678} &= 0 \\ \omega_8^{86} \gamma_6 + \omega_8^{79} \gamma_{79}^8 + e^\Phi H_{0678} \gamma^{0678} &= 0 \\ \omega_9^{96} \gamma_6 + \omega_9^{78} \gamma_{78}^9 - e^\Phi H_{0678} \gamma^{0678} &= 0 \end{aligned}$$

Some of these equations determine the field strengths in terms of the spin connections, to be

$$e^\Phi H_{078a} = -\frac{1}{2}(\omega_0^{0a} + \omega_7^{7a})$$

$$\begin{aligned}
e^\Phi H_{068a} &= \frac{1}{2}\omega_6^{a7} \\
e^\Phi H_{067a} &= -\frac{1}{2}\omega_6^{a8} \\
e^\Phi H_{0678} &= -\omega_0^{06} \\
e^\Phi G_{069a} &= \frac{1}{2}(\omega_0^{0a} - \omega_7^{7a}) \\
e^\Phi G_{09a\alpha} &= \frac{1}{2}\omega_6^{a\alpha} \\
e^\Phi G_{069\alpha} &= \omega_0^{0\alpha}
\end{aligned} \tag{11}$$

The remaining equations yield further constraints on the metric.

$$\begin{aligned}
\omega_1^{67}\gamma_{67}^1 &= \omega_1^{89}\gamma_{89}^1 \\
\omega_6^{17}\gamma_{17}^6 &= -\omega_7^{16}\gamma_{16}^7 = \omega_8^{19}\gamma_{19}^8 = -\omega_9^{18}\gamma_{18}^9 \\
\omega_6^{98}\gamma_{98}^6 &= (\omega_0^{07} - \omega_6^{67})\gamma_7 \\
\omega_8^{96}\gamma_{96}^8 &= (-\omega_0^{07} - \omega_8^{87})\gamma_7 \\
\omega_9^{68}\gamma_{68}^9 &= (\omega_0^{07} - \omega_9^{97})\gamma_7 \\
(\omega_7^{76} + \omega_9^{96})\gamma_6 &+ \omega_7^{89}\gamma_{89}^7 + \omega_9^{78}\gamma_{78}^9 = 0 \\
\omega_7^{89}\gamma_{89}^7 &= \omega_8^{79}\gamma_{79}^8 \\
\omega_7^{89}\gamma_{89}^7 &= (\omega_0^{06} - \omega_8^{86})\gamma_6
\end{aligned} \tag{12}$$

C. Solution

We can solve the constraints (8,12) by the metric ansatz

$$\begin{aligned}
e_{0\tilde{0}} &= \lambda^{-\frac{1}{4}} & e_{a\tilde{a}} &= \lambda^{\frac{1}{4}} & e_{6\tilde{6}} &= \lambda^{-\frac{1}{4}}H^{\frac{1}{2}} \\
e_{7\tilde{7}} &= e_{8\tilde{8}} = \lambda^{\frac{1}{4}}H^{\frac{1}{2}} & e_{9\tilde{9}} &= \lambda^{-\frac{1}{4}}H^{-\frac{1}{2}} \\
e_{9\tilde{7}} &= e_{9\tilde{9}}A_7 & e_{9\tilde{8}} &= e_{9\tilde{9}}A_8 & e_{6\tilde{a}} &= e_{6\tilde{6}}\phi_a
\end{aligned} \tag{13}$$

with the remaining constraints

$$\begin{aligned}
\partial_7 A_8 - \partial_8 A_7 &= -\partial_6(H\lambda) \\
\partial_6 A_8 &= \partial_7 H & \partial_6 A_7 &= -\partial_8 H \\
\partial_6(\phi_a H) &= \partial_a H
\end{aligned} \tag{14}$$

Similarly by requiring the variation of the dilatino to vanish, we find that the dilaton is given by

$$e^\Phi = \lambda^{\frac{1}{4}} \tag{15}$$

We can then show that the field strengths (6,11) can be obtained from the gauge fields

$$\begin{aligned}
A_{09a} &= -\frac{1}{4} \frac{\phi_a}{\lambda} & A_{069} &= \frac{1}{4\lambda} \\
A_{06\alpha} &= \frac{A_\alpha}{4\lambda} & A_{0a\alpha} &= \frac{A_\alpha \phi_a}{4\lambda} \\
A_{078} &= -\frac{H}{4}
\end{aligned} \tag{16}$$

Finally we impose the equations of motion for pointlike sources. These yield the equation

$$\partial_a \phi_a = H^{-1} \partial_6 \lambda + \partial_6 \left(\frac{\phi_a^2}{2} \right) \tag{17}$$

This provides a complete solution for the 2-brane on a 2-cycle of a Z_N orbifold.

D. Adding a B-field

In the solution of the previous section, we were at the point in moduli space where the B-field on the orbifold was zero, as could be seen from the fact that the D0-brane charge was zero. Turning on a B-field on the orbifold point will add zero-brane charge to the system.

To find a solution with added zero-brane charge, we lift the solution we have found to 11 dimensions (thus getting a M2brane on the orbifold.) We then boost in the 11th direction and dimensionally reduce, thus obtaining a solution with D2+D0 charge.

All these steps are straightforward, and we can directly present the final answer in the next section. The solutions for other branes on the orbifold can be obtained by T-duality.

III. BRANES ON ORBIFOLDS

A. Notation

First of all we summarize the various formulae that are required in the solutions.

The solutions are expressed in terms of the functions λ, H, A_7, A_8 . In addition we will introduce a constant β and a function Δ defined as

$$\Delta = \cosh^2 \beta - \lambda^{-1} \sinh^2 \beta \quad (18)$$

The constant β controls the value of the B-field on the 2-cycle. $\beta = 0$ corresponds to zero B-field.

The functions satisfy the differential equations

$$\begin{aligned} \partial_7 A_8 - \partial_8 A_7 + \partial_6 (H\lambda) &= Q(x_a, x_6) \delta(x_\alpha) \\ \partial_6 A_8 &= \partial_7 H & \partial_6 A_7 &= -\partial_8 H \\ \partial_6 (\phi_a H) &= \partial_a H \\ \partial_a \phi_a &= H^{-1} \partial_6 \lambda + \partial_6 \left(\frac{\phi_a^2}{2} \right) \end{aligned} \quad (19)$$

These functions can be expressed in terms of a single function τ through

$$H\phi_a = \partial_a \partial_6 \tau \quad (20)$$

$$H = \partial_6^2 \tau \quad (21)$$

$$\lambda + H\phi_a^2 = \partial_a^2 \tau \quad (22)$$

τ then satisfies the differential equation

$$\partial_a^2 \tau + \partial_6^2 \partial_a^2 \tau - (\partial_6 \partial_a \tau)^2 = \frac{1}{\partial_6} Q(x_a, x_6) \delta(x_\alpha) \quad (23)$$

The function Q parametrizes the brane source; in particular, it incorporates the effects of brane bending.

We will denote the NSNS 2-form by $B_{\mu\nu}$, and the RR forms by $C_{\mu_1 \dots \mu_k}^{(k)}$.

B. 2-brane on Z_N orbifold

Here a runs from 1 to 5. The metric is

$$ds^2 = \lambda^{-\frac{1}{2}} \Delta^{-\frac{1}{2}} (-dt^2 + \Delta dx_a^2) + \lambda^{-\frac{1}{2}} \Delta^{\frac{1}{2}} H (dx^6 + \phi_a dx^a)^2 + \lambda^{\frac{1}{2}} \Delta^{\frac{1}{2}} H (dx_7^2 + dx_8^2) + \lambda^{-\frac{1}{2}} \Delta^{\frac{1}{2}} H^{-1} (dx_9 + A_7 dx^7 + A_8 dx_8)^2 \quad (24)$$

The dilaton is

$$e^\Phi = \lambda^{\frac{1}{4}} \Delta^{\frac{3}{4}} \quad (25)$$

The gauge fields are

$$\begin{aligned} C_0^{(1)} &= \sinh\beta \cosh\beta \frac{(1-\lambda)}{\Delta\lambda} \\ C_{09a}^{(3)} &= -\frac{1}{4} \frac{\phi_a}{\lambda} \cosh\beta & C_{069}^{(3)} &= \frac{1}{4\lambda} \cosh\beta \\ C_{06\alpha}^{(3)} &= \frac{A_\alpha}{4\lambda} \cosh\beta & C_{0a\alpha}^{(3)} &= \frac{A_\alpha \phi_a}{4\lambda} \cosh\beta \\ C_{078}^{(3)} &= -\frac{H}{4} \cosh\beta \\ B_{9a} &= -\frac{3}{8} \frac{\phi_a}{\lambda} \sinh\beta & B_{69} &= \frac{3}{8\lambda} \sinh\beta \\ B_{6\alpha} &= \frac{3A_\alpha}{8\lambda} \sinh\beta & B_{a\alpha} &= \frac{3A_\alpha \phi_a}{8\lambda} \sinh\beta \\ B_{78} &= -\frac{3H}{8} \sinh\beta \end{aligned} \quad (26)$$

C. 3-brane on Z_N orbifold

Here i runs over 0, 1, a runs over 2, 3, 4, 5. The metric is

$$ds^2 = \lambda^{-\frac{1}{2}} \Delta^{-\frac{1}{2}} (-dt^2 + dx_1^2 + \Delta dx_a^2) + \lambda^{-\frac{1}{2}} \Delta^{\frac{1}{2}} H (dx^6 + \phi_a dx^a)^2 + \lambda^{\frac{1}{2}} \Delta^{\frac{1}{2}} H (dx_7^2 + dx_8^2) + \lambda^{-\frac{1}{2}} \Delta^{\frac{1}{2}} H^{-1} (dx_9 + A_7 dx^7 + A_8 dx_8)^2 \quad (27)$$

The dilaton is

$$e^\Phi = \Delta^{\frac{1}{2}} \quad (28)$$

The gauge fields are

$$\begin{aligned}
C_{01}^{(2)} &= \sinh\beta \cosh\beta \frac{(1-\lambda)}{\Delta\lambda} \\
C_{abc7}^{(4)} &= \frac{1}{4}\epsilon_{abcd}(\phi_d A_7 + \partial_8 \partial_d \tau) & C_{abc8}^{(4)} &= \frac{1}{4}\epsilon_{abcd}(\phi_d A_8 - \partial_7 \partial_d \tau) \\
C_{abc9}^{(4)} &= \frac{1}{4}\epsilon_{abcd}\phi_d \\
B_{9a} &= -\frac{3}{8}\frac{\phi_a}{\lambda}\sinh\beta & B_{69} &= \frac{3}{8\lambda}\sinh\beta \\
B_{6\alpha} &= \frac{3A_\alpha}{8\lambda}\sinh\beta & B_{a\alpha} &= \frac{3A_\alpha\phi_a}{8\lambda}\sinh\beta \\
B_{78} &= -\frac{3H}{8}\sinh\beta
\end{aligned}$$

D. 4-brane on Z_N orbifold

Here i runs over 1, 2, a runs over 3, 4, 5. The metric is

$$\begin{aligned}
ds^2 &= \lambda^{-\frac{1}{2}}\Delta^{-\frac{1}{2}}(-dt^2 + dx_i^2 + \Delta dx_a^2) + \lambda^{-\frac{1}{2}}\Delta^{\frac{1}{2}}H(dx^6 + \phi_a dx^a)^2 + \lambda^{\frac{1}{2}}\Delta^{\frac{1}{2}}H(dx_7^2 + dx_8^2) \\
&\quad + \lambda^{-\frac{1}{2}}\Delta^{\frac{1}{2}}H^{-1}(dx_9 + A_7 dx^7 + A_8 dx_8)^2 \quad (29)
\end{aligned}$$

The dilaton is

$$e^\Phi = \lambda^{-\frac{1}{4}}\Delta^{\frac{1}{4}} \quad (30)$$

The gauge fields are

$$\begin{aligned}
C_{012}^{(3)} &= \sinh\beta \cosh\beta \frac{(1-\lambda)}{\Delta\lambda} \\
C_{ab7}^{(3)} &= \frac{1}{4}\epsilon_{abc}(\phi_c A_7 + \partial_8 \partial_c \tau) & C_{ab8}^{(3)} &= \frac{1}{4}\epsilon_{abc}(\phi_c A_8 - \partial_7 \partial_c \tau) \\
C_{ab9}^{(3)} &= \frac{1}{4}\epsilon_{abc}\phi_c \\
B_{9a} &= -\frac{3}{8}\frac{\phi_a}{\lambda}\sinh\beta & B_{69} &= \frac{3}{8\lambda}\sinh\beta \\
B_{6\alpha} &= \frac{3A_\alpha}{8\lambda}\sinh\beta & B_{a\alpha} &= \frac{3A_\alpha\phi_a}{8\lambda}\sinh\beta \\
B_{78} &= -\frac{3H}{8}\sinh\beta
\end{aligned}$$

E. 5-brane on Z_N orbifold

Here i runs over 1, 2, 3, a runs over 4, 5. The metric is

$$ds^2 = \lambda^{-\frac{1}{2}} \Delta^{-\frac{1}{2}} (-dt^2 + dx_i^2 + \Delta dx_a^2) + \lambda^{-\frac{1}{2}} \Delta^{\frac{1}{2}} H (dx^6 + \phi_a dx^a)^2 + \lambda^{\frac{1}{2}} \Delta^{\frac{1}{2}} H (dx_7^2 + dx_8^2) + \lambda^{-\frac{1}{2}} \Delta^{\frac{1}{2}} H^{-1} (dx_9 + A_7 dx^7 + A_8 dx_8)^2 \quad (31)$$

The dilaton is

$$e^\Phi = \lambda^{-\frac{1}{2}} \quad (32)$$

The gauge fields are

$$\begin{aligned} C_{0123}^{(4)} &= \sinh\beta \cosh\beta \frac{(1-\lambda)}{\Delta\lambda} \\ C_{a7}^{(2)} &= \frac{1}{4} \epsilon_{ab} (\phi_b A_7 + \partial_8 \partial_b \tau) & C_{a8}^{(2)} &= \frac{1}{4} \epsilon_{ab} (\phi_b A_8 - \partial_7 \partial_b \tau) \\ C_{a9}^{(2)} &= \frac{1}{4} \epsilon_{ab} \phi_b \\ B_{9a} &= -\frac{3}{8} \frac{\phi_a}{\lambda} \sinh\beta & B_{69} &= \frac{3}{8\lambda} \sinh\beta \\ B_{6\alpha} &= \frac{3A_\alpha}{8\lambda} \sinh\beta & B_{a\alpha} &= \frac{3A_\alpha \phi_a}{8\lambda} \sinh\beta \\ B_{78} &= -\frac{3H}{8} \sinh\beta \end{aligned}$$

IV. COMMENTS ON THE DUALITY

The last subsection above describes the supergravity solution for 5-branes wrapped on an orbifold 2-cycle. By taking the near-horizon limit, we obtain the dual to $\mathcal{N} = 2$ gauge theory in 3+1 dimensions.

The orbifold point is defined by the point in the moduli space where $B = \frac{1}{N}$ for a Z_N orbifold. At this point in moduli space, a single 2-brane wrapped on the orbifold gets an induced 0-brane charge equal to $\frac{1}{N}$. This can be used to fixed the value of β at the orbifold point.

The calculation is straightforward, given the supergravity solution. We find that (we set $\alpha' = 1$)

$$\sinh\beta = \frac{1}{4\pi Na} \quad (33)$$

(Recall that a is the separation between the centers in the orbifold metric (2).) This value of β is the perturbative orbifold point. Other values of β correspond to changing the B-field on the 2-cycle.

One should also ask where the supergravity solution we have found is valid. For this, we need the string coupling to be small. Hence

$$e^\Phi = g_0 \lambda^{-\frac{1}{2}} \ll 1 \quad (34)$$

This means that the supergravity solution cannot be trusted near the branes. This corresponds to the fact that the gauge theory is trivial in the infrared, and cannot therefore have a simple description in supergravity.

Also, we need the curvature to be small. We cannot find the exact regime where this applies, since we have not found an analytic solution. However, we know that this must break down far away from the branes, since asymptotically, the geometry is similar to that of a five-brane. This breakdown corresponds to the fact that the gauge theory is asymptotically free, and again there cannot be a good supergravity description.

Finally, we can see that in the weak coupling region, the gauge coupling has a logarithmic falloff with scale, as is expected from the gauge theory correspondence. This can be seen from the fact that the gauge coupling asymptotically satisfies the Laplace equation in 2 space dimensions, which has a logarithmic solution. This analysis is the same as that performed in the geometric engineering analyses of field theories [15–17].

V. CONCLUSION

In summary, we have constructed supergravity duals for $\mathcal{N} = 2$ gauge theories. This method can be extended to all theories which can be obtained on branes through geometric

engineering. This is a very wide class of theories.

We have left the detailed analysis of the correspondence to future work. It would be interesting to see what information can be obtained about the gauge theories by this duality. In particular, it should be possible to rederive the results of [18,19]. It should also be possible to analyze the behaviour of solitons in these theories by analyzing branes ending on other branes.

VI. ACKNOWLEDGEMENTS

This research was supported in part by DOE grant DE-FG02-96ER40559.

REFERENCES

- [1] J. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [hep-th/9711200].
- [2] J. Polchinski and M. J. Strassler, hep-th/0003136.
- [3] I. R. Klebanov and A. A. Tseytlin, Nucl. Phys. **B578**, 123 (2000) [hep-th/0002159].
- [4] I. R. Klebanov and M. J. Strassler, JHEP **0008**, 052 (2000) [hep-th/0007191].
- [5] B. S. Acharya, hep-th/0011089.
- [6] J. M. Maldacena and C. Nunez, hep-th/0008001.
- [7] M. Atiyah, J. M. Maldacena and C. Vafa, hep-th/0011256.
- [8] B. Brinne, A. Fayyazuddin, S. Mukhopadhyay and D. J. Smith, hep-th/0009047.
- [9] E. Witten, Nucl. Phys. Proc. Suppl. **68**, 216 (1998).
- [10] A. Rajaraman, hep-th/0007241.
- [11] M. R. Douglas and G. Moore, hep-th/9603167.
- [12] C. V. Johnson and R. C. Myers, Phys. Rev. **D55**, 6382 (1997) [hep-th/9610140].
- [13] P. S. Aspinwall, Phys. Lett. **B357**, 329 (1995) [hep-th/9507012].
- [14] J. Polchinski, hep-th/0011193.
- [15] S. Katz and C. Vafa, Nucl. Phys. **B497**, 196 (1997) [hep-th/9611090].
- [16] S. Katz, A. Klemm and C. Vafa, Nucl. Phys. **B497**, 173 (1997) [hep-th/9609239].
- [17] S. Katz, P. Mayr and C. Vafa, Adv. Theor. Math. Phys. **1**, 53 (1998) [hep-th/9706110].
- [18] N. Seiberg and E. Witten, Nucl. Phys. **B426**, 19 (1994) [hep-th/9407087].
- [19] M. R. Douglas and S. H. Shenker, Nucl. Phys. **B447**, 271 (1995) [hep-th/9503163].